Michigan State University
January 10, 2025

Course Details

- **Instructor:** Son Tu - tuson@msu.edu.
- **Office:** C330 Wells Hall.
- **Office Hours:** Monday from 1:00pm – 2:00pm, Friday 1:00 - 2:00 pm, and by appointment.
- **Website:** `https://users.math.msu.edu/users/tuson/` and **D2L:** `https://d2l.msu.edu/d2l/home/2129429`.
- **Course Description:** This course introduces the optimal control problem, delving into its fundamental principles and its link to the Hamilton-Jacobi-Bellman equation. We will delve into the interaction between nonlinear optimization and control problems, exploring various facets of the theory, including dynamic programming, variational calculus, and Pontryagin's maximum principle. The curriculum incorporates numerous examples and applications of the theory within continuous systems. Towards the conclusion of the course, attention will shift to the study of numerical search algorithms and, time permitting, discrete systems.
- **Required background:** Recommended background: CSE 231 or CMSE 201. Prerequisites: MTH 235 or MTH 340 and department approval.
- **Course Schedule: Lectures:** Monday, Wednesday, and Friday, 11:30 AM–12:20 PM, in A222 Wells Hall.
- **Text:** There is no required textbook for this course. Lecture notes will be distributed after each class.
- **Attendance:** Attendance is not required but is STRONGLY encouraged.
- **Final Exam: Thursday, May 1, 2025:** 12:45 PM–2:45 PM (accounting for 20% of your grade).
- **Midterm:** There will be one midterm exam, conducted during class, accounting for 20% of your grade.
- **Homework:** There will be approximately 6–8 homework assignments throughout the course. The average of these grades will determine your total homework grade, which will account for 60% of your overall course grade. Homework will be assigned on Fridays, with two weeks given to complete each set.
- **Final Project:** The final project will include a written report, accounting for 20% of your total grade. This will be treated as extra credit.
- **Grading Scale:** Your total grade will be calculated as a weighted average of the following: Homework: 60%, Midterm: 20%, Final Exam: 20%. The grading scale will be no stricter than the following:

Percentage (P) %	Grade
$90 \leq P$	4.0
$85 \leq P < 90$	3.5
$80 \leq P < 85$	3.0
$75 \leq P < 80$	2.5
$70 \leq P < 75$	2.0
$65 \leq P < 70$	1.5
$60 \leq P < 65$	1.0
$P < 60$	0

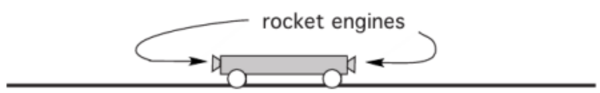
- **Academic Accommodations:** If you require academic accommodations through a disability or ailment as deemed by the RCPD office (Resource Center for Persons with Disabilities), please bring us your visa as soon as possible. If you feel that you are qualified for an accommodation, but you have not already done so, please visit the RCPD office immediately.

You may schedule a visitation or find other information out about the office or the accommodations in general by visiting: `https://www.rcpd.msu.edu`.
You may also visit the office directly at: **Bessey Hall, 434 Farm Lane, #120**.
Or by calling: `(517)884-RCPD(4-7273)`

- **Academic Honesty:** For full details please visit: `https://www.msu.edu/~ombud/academic-integrity/index.html`. *The straightforward rule is: DO NOT CHEAT.* This includes, but is not limited to, copying another student's work during an exam or using electronic devices to gain an unfair advantage. Violations may result in receiving a zero on the exam, a failing grade for the course, or even expulsion from the University. Simply put, do not cheat and avoid putting yourself in that situation.

Introduction

Imagine a railroad car powered by rocket engines on each side.



We introduce the variables

- $x(t)$ is the position of the rocket railroad car on the train track at time t
- $v(t)$ is the velocity of the rocket rail road car at time t
- The force from the rocket engines at time t, where we only consider $F(t) \in [-1, 1]$, and the sign of $F(t)$ depends on which engine is firing. firing.

One might ask, starting at a given location A, whether there exists a choice F so that the car stops at a predetermined location B. If so, is there a way to do so with minimal time or energy?
This constitutes the basic problem of optimal control theory. Here is a quote from Wikipedia regarding Richard E. Bellman:

> *In 1949, Bellman worked for many years at RAND corporation, and it was during this time that he developed dynamic programming.*

The so-called Dynamic Programming Principle connects the control problem above to a partial differential equation known as the Hamilton–Jacobi–Bellman equation. We will provide an introduction to this topic and explore the fascinating connection between dynamics and partial differential equations.

Outline (subject to time constraints)

1. Introduction to Optimal Control Theory.
   What are the systems where the optimal control problems arises, and naively what can we show them?
2. Linear Control systems.
   Objective: Understand the systematic study of optimization problems that lead to the formulation of optimal control.
   Content: Focus on linear systems where solutions are straightforward to solve.
3. Introduction to Calculus of variations.
   ODE theory review.
4. Pontryagin Maximum Principle.
5. Introduction to Dynamic Programming Principle and Hamilton-Jacobi equations.
6. Some numerical algorithms to solve optimal control problems.
   Matlab or Python.
7. Some applications.
   Shortest distance on a map, economics applications, ...

References

In case you want to have some references, I suggest some followings (in no particular order):

1. Optimal Control Applied to Biological Models, By Suzanne Lenhart and John T. Workman.

2. A lecture note by Alberto Bressan and Benedetto Piccoli:
   Introduction to the Mathematical Theory of Control.

3. A book by Hung V. Tran:
   Hamilton–Jacobi Equations: Theory and Applications It is available free of charge at
   `http://math.wisc.edu/ hung/HJ-equations-Tran-AMS.pdf`.

4. Chapter 10 of Lawrence C. Evans's PDEs book:
   Partial Differential Equations: Second Edition

5. A lecture note by Khai T. Nguyen
   Topics on optimal control and PDEs
   It is available free of charge at
   `https://tnguye13.math.ncsu.edu/course1.pdf`.