Michigan State University
Math 234 – Spring 2024

Practice Exam 02 and some more problems
2024-03-05

Disclaimer: This exam was created from previous semesters' exams. The problems given on the old exams are not necessarily typical, and the current semester's exams may differ significantly from these ones.

1. Consider $f(x, y) = \frac{1}{\sqrt{y-x^2-1}}$, what is the domain and the range of f?

2. Consider the parametric surface

$$\mathbf{r}(u, v) = (u^2 + 1, v^3 + 1, u + v).$$

   Find the normal vector at $(2, 2, 0)$ and the tangent plane at that point to the surface.

3. Find the absolute maximum of $f(x, y) = x + y - xy$ on the closed triangular region with vertices $(0, 0), (0, 2)$ and $(4, 0)$.

4. Let $f(x, y) = \frac{x}{x+y}$, find the linearization of f at $(2, 1)$. Use your answer to approximate $f(2.2, 0.9)$. What is the minimum rate of change of f at $(2, 1)$?

5. Find the work done by the force $F = (y, -xy)$ along the straight-line segment from $(0, 0)$ to $(3, 1)$.

6. Let C be the curve whose parametrization is given by the vector equation $r(t) = (2t, t)$, $0 \leqslant t \leqslant 1$. Find

$$\int_C (xy + y)\, ds.$$

7. Let C be the union of the line segments from $P(2, 0, \pi)$ to $R(0, 0, 0)$ and from R to $Q(1, 1, \pi/2)$. Evaluate

$$\int_C (2x + 5y^2 z)\, dx + (10xyz - 3e^{3y} \cos z)\, dy + (5xy^2 + e^{3y} \sin z)\, dz$$

8. Find the value of $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ at the point $(1, 1, 1)$ if z as a function of x and y is defined by the equation $z^3 - xy + yz + y^3 - 2 = 0$.

9. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint

$$f(x, y) = 3x + y, \qquad x^2 + y^2 = 10.$$

   Write down the values of possible λ.

10. Let $z = f(x, y)$, where f is differentiable, and $x = g(t)$ and $y = h(t)$. Suppose that $g(3) = 2$, $h(3) = 7$, $g'(3) = 5$, $h'(3) = -4$, $f_x(2, 7) = 6$ and $f_y(2, 7) = -8$. Find $\frac{dz}{dt}$ when $t = 3$.

11. Consider the function $f(x,y) = x^2 + xy + y^2$ at the point $(-1,1)$. In what direction does f decrease most rapidly?

12. True of False? $f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$

13. True of False? $D_{\mathbf{k}} f(x,y,z) = f_z(x,yz)$?

14. True of False? If f has a local minimum at $(1,2)$ and f is differentiable at $(1,2)$ then $\nabla f(1,2) = 0$.

15. True of False? If $f(x,y)$ has two local maxima, then f must have at least one local minimum?

16. True of False? If f has continuous partial derivatives on and C is any circle then

$$\int_C \nabla f \cdot d\mathbf{r} = 0.$$

17. Which of the following vector field plots could be $F = (xy, -y)$?