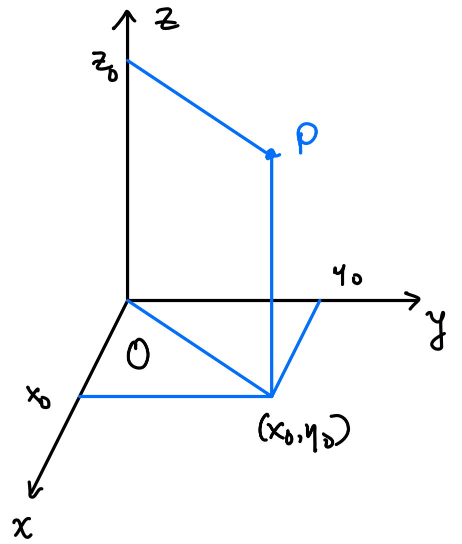
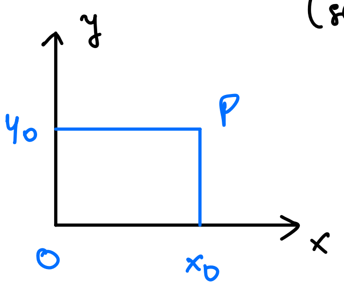
The 3-dimensional coordinate system



a point $P(x_0, y_0, z_0)$
↪ coordinate

The projection to xy-plane
(set $z = 0$)



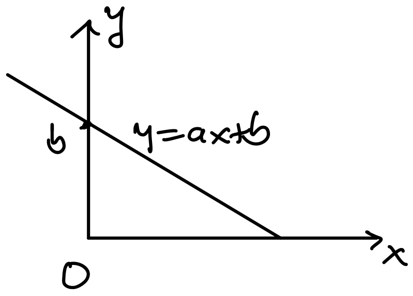
$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
$= \{ (x, y, z) : x, y, z \in \mathbb{R} \}$

ordered-dripple

Line in 2D: $ax + by + c = 0$, or $y = \underset{\substack{\uparrow \\ \text{slope}}}{a}x + \underset{\substack{\nwarrow \\ \text{y-intercept}}}{b}$

ex. check if $(1, 4)$ is on the line $x - 4y = 1$

Ans : $1 - 4.4 \neq 1$: No



Plane in 3D: $ax + by + cz + d = 0$

ex: check if $(1, 4, 2)$ is on the plane $x - 4y + 8z = 1$

Ans $1 - 4.4 + 8.2 = 1$ Yes

Surface in 3D

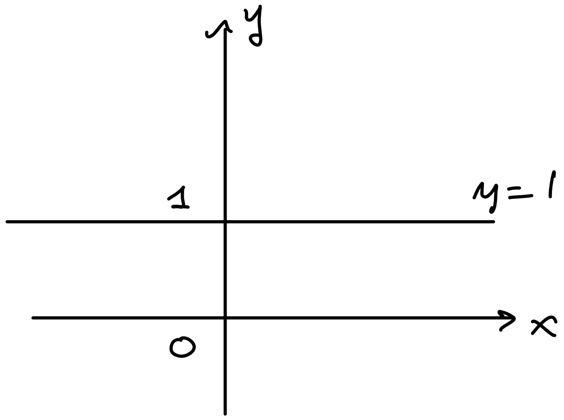
ex: check if $(1, -3, 0)$ is on the surface $xyz + x^2 = y$

Ans : $0 + 1 = -3$ No

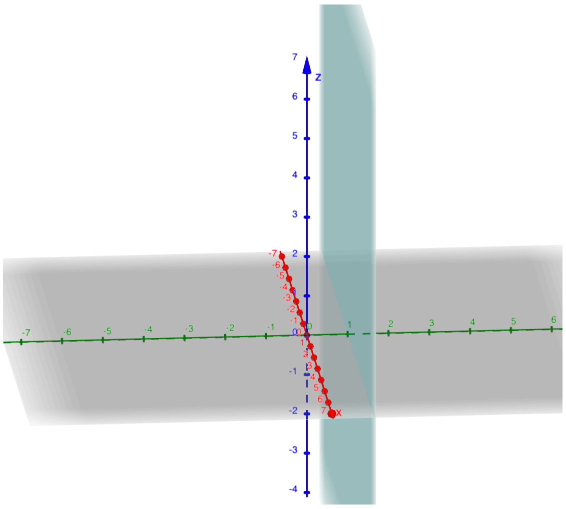
Note : projection of plane $(z = 0)$ to xy-plane is a line: $ax + by + d = 0$ in 2D

Graph:

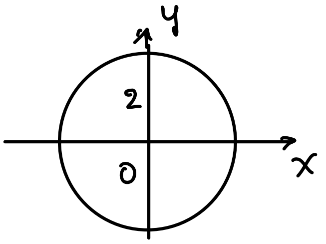
Ex. graph $y = 1$ on xy-plane



Ex: graph $y = 1$ in $\mathbb{R}^3$



Ex. graph $x^2+y^2=4$ in xy-plane



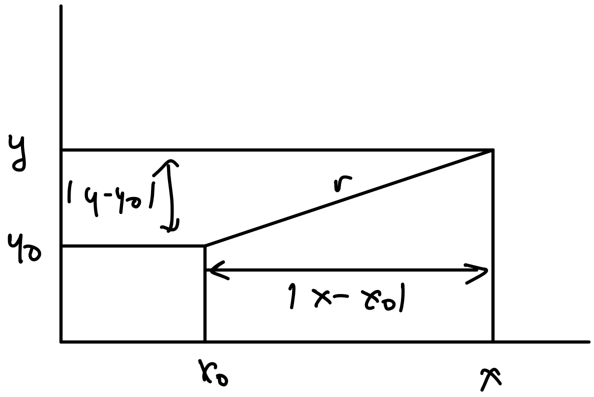
circle radius 2

centered at (0,0)

general form:

$(x-x_0)^2+(y-y_0)^2=r^2$ → circle centered at (x_0, y_0), radius r

↳ collection of all points (x,y) whose distance to (x_0, y_0) is r



$r^2 = (x-x_0)^2+(y-y_0)^2$

Pythagorean

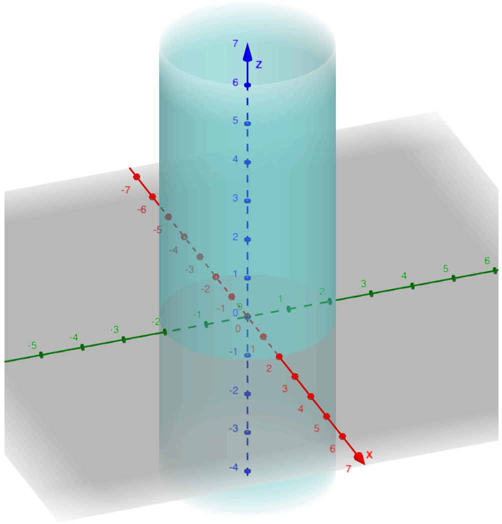
Distance of a point $P(x,y,z)$ to $P_0(x_0,y_0,z_0)$ in 3D

$$|PP_0| = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

Ex: graph $x^2+y^2=4$ in 3D

Note: projection to xy-plane is exactly the circle $x^2+y^2=4$

cylinder

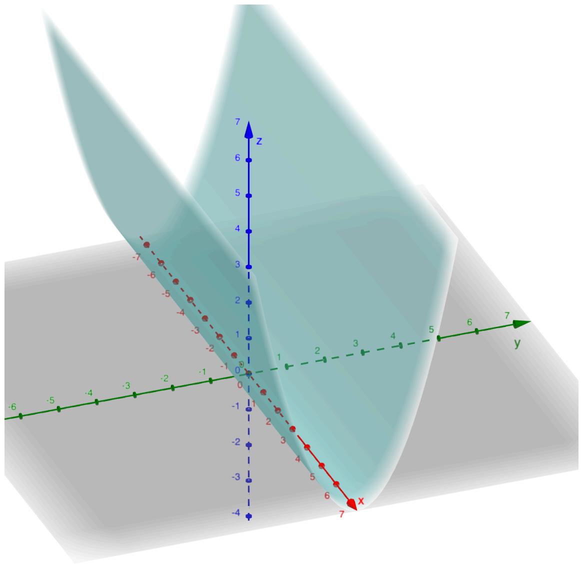


Ex graph $x^2+y^2+z^2=4$ in 3D

↳ sphere

Ex: graph $z=y^2$ in $\mathbb{R}^3$

also cylinder



Ex: $x^2-2x+y^2+z^2+4z=4$

↳ complete the square

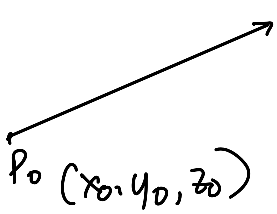
$$(x^2-2x+1)+y^2+(z^2+4z+4)=1+4+4=9$$

$$(x-1)^2+y^2+(z+2)^2=3^2$$

↳ sphere centered at $(1,0,-2)$, radius 3

Vectors : a vector is a dripple (a, b, c)

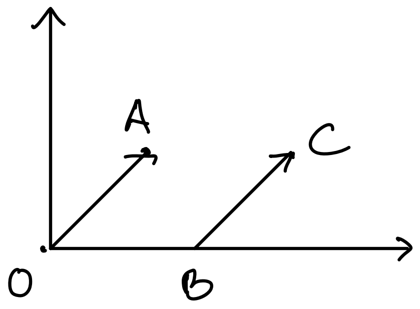
a vector $\overrightarrow{P_0P}$ has direction



$\overrightarrow{P_0P}$ = (end) - (start) = $(x, y, z) - (x_0, y_0, z_0)$

a vector depends only < direction, length

for example



O (0,0)

A (2,2)

B (3,0)

C (5,2)

$\overrightarrow{OA} = (2,2)$, $\overrightarrow{BC} = (2,2)$

so $\overrightarrow{OA} = \overrightarrow{BC}$

only depends on { direction, length !

length $|\overrightarrow{PP_0}| = |\overrightarrow{P_0P}| = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$

distance between P, P_0

or if $\vec{v} = (a, b, c)$

then $|\vec{v}| = \sqrt{a^2+b^2+c^2}$