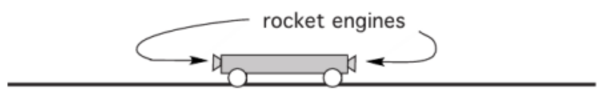
## Introduction

Imagine a railroad car powered by rocket engines on each side.




We introduce the variables

- $x(t)$ is the position of the rocket railroad car on the train track at time $t$
- $v(t)$ is the velocity of the rocket rail road car at time $t$
- The force from the rocket engines at time $t$, where we only consider $F(t) \in [-1, 1]$, and the sign of $F(t)$ depends on which engine is firing. firing.

One might ask, starting at a given location $A$, whether there exists a choice $F$ so that the car stops at a predetermined location $B$. If so, is there a way to do so with minimal time or energy?
This constitutes the basic problem of optimal control theory. Here is a quote from Wikipedia regarding Richard E. Bellman:

> *In 1949, Bellman worked for many years at RAND corporation, and it was during this time that he developed dynamic programming.*

The so-called Dynamic Programming Principle connects the control problem above to a partial differential equation known as the Hamilton–Jacobi–Bellman equation. We will provide an introduction to this topic and explore the fascinating connection between dynamics and partial differential equations.

## Instructor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## Course Description

We will delve into the foundational aspects of the following related subjects:

- An introduction to the optimal control problem and the first-order Hamilton-Jacobi-Bellman equation.
- An overview of viscosity solutions and fundamental techniques.
- (If time allows) Engaging in projects centered around one-dimensional examples and enhancing research-level findings from the existing literature.

The anticipated commitment for this reading plan is around 1 credit, involving approximately 1-1.5 hours of weekly meetings, complemented by additional reading at home. Our main aim is to have fun exploring and appreciate the beauty of viscosity solutions in optimal control theory.

## Additional topics might be covered

- Homogenization theory.
- Numerical computation of optimal control and viscosity solution.

## Required background

- Linear algebra
- Some basic real analysis and measure theory, Ordinary differential equations will be sufficient.

## References

In case you want to have some references, I suggest some followings (in no particular order):

1. A lecture note by Alberto Bressan and Benedetto Piccoli:
   *Introduction to the Mathematical Theory of Control.*
   It is available free of charge at
   `https://www-aimsciences-org.proxy2.cl.msu.edu/book/AM/volume/27`
2. A book by Hung V. Tran:
   *Hamilton–Jacobi Equations: Theory and Applications, volume 213 of Graduate studies in Mathematics. American Mathematical Society, 2021.*
   It is available free of charge at
   `http://math.wisc.edu/ hung/HJ-equations-Tran-AMS.pdf`.
3. Chapter 10 of Lawrence C. Evans's PDE book:
   *Partial Differential Equations: Second Edition*
4. A lecture note by Khai T. Nguyen
   *Topics on optimal control and PDEs*
   It is available free of charge at
   `https://tnguye13.math.ncsu.edu/course1.pdf`.