Errata

Han, Y., and Tu, S. N. T. Remarks on the Vanishing Viscosity Process of State-Constraint Hamilton–Jacobi Equations. *Applied Mathematics & Optimization 86*, 1 (June 2022), 3

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We made a typo in the statement of [1, Lemma 18], which led to the lower rate $\mathcal{O}(\varepsilon^{1/p})$ as recorded in [1, Corollary 3]. The corrected version yields a better rate $\mathcal{O}\left(\varepsilon^{1/(p-\frac{1}{2})}\right)$. The proof in [1, Lemma 18] was written for the corrected version.

The correct statement is $\|f_\kappa - f\|_{L^\infty(\Omega)} \leqslant C\kappa^2$, not $\|f_\kappa - f\|_{L^\infty(\Omega)} \leqslant C\kappa$ as stated in [1, Lemma 18].

Lemma 18. Assume $f \in C^2(\overline{\Omega})$ such that $f = 0$ and $Df = 0$ on $\partial\Omega$. For all $\kappa > 0$ small enough, there exists $f_\kappa \in C^2_c(\Omega)$ such that

$$\|f_\kappa - f\|_{L^\infty(\Omega)} \leqslant C\kappa^2 \qquad \text{and} \qquad \|D^2 f_\kappa\|_{L^\infty(\Omega)} \leqslant C$$

where C is independent of κ.

Later, in Corollary 3, the use of the incorrect statement led to a slower rate $\mathcal{O}(\varepsilon^{1/p})$. Here, we correct it and provide the improved rate that Corollary 3 should have obtained.

Proof of Corollary 3. Let $u^\varepsilon_\kappa \in C^2(\Omega) \cap C(\overline{\Omega})$ be the solution to (PDE$_\varepsilon$) and u_k be the solution to (PDE$_0$) with f replaced by f_k, respectively. It is clear that

$$0 \leqslant u^\varepsilon(x) - u^\varepsilon_\kappa(x) \leqslant C\kappa^2 \qquad \text{for } x \in \Omega$$

and

$$0 \leqslant u(x) - u_\kappa(x) \leqslant C\kappa^2 \qquad \text{for } x \in \Omega.$$

Therefore,

$$u^\varepsilon(x) - u(x) \leqslant 2C\kappa^2 + \left(u^\varepsilon_\kappa(x) - u_\kappa(x)\right). \tag{57}$$

By Theorem 2 and Remark 8, as $f_\kappa \in C^2_c(\Omega)$ with a uniform bound on $D^2 f_\kappa$, we have

$$u^\varepsilon_\kappa(x) - u_\kappa(x) \leqslant \frac{\nu C_\alpha \varepsilon^{\alpha+1}}{d(x)^\alpha} + C\left(\left(\frac{\varepsilon}{\kappa}\right)^{\alpha+1} + \left(\frac{\varepsilon}{\kappa}\right)^{\alpha+2}\right) + 4nC\varepsilon, \qquad p < 2,$$

$$u^\varepsilon_\kappa(x) - u_\kappa(x) \leqslant \nu\varepsilon \log\left(\frac{1}{d(x)}\right) + C\left(\left(\frac{\varepsilon}{\kappa}\right) + \left(\frac{\varepsilon}{\kappa}\right)^2\right) + 4nC\varepsilon, \qquad p = 2$$

for some constant C independent of κ. Choose $\kappa = \varepsilon^\gamma$ with $\gamma \in (0,1)$. Then (57) becomes

$$u^\varepsilon(x) - u(x) \leqslant C\varepsilon^{2\gamma} + C\varepsilon + \frac{C\varepsilon^{\alpha+1}}{d(x)^\alpha} + C\varepsilon^{(1-\gamma)(\alpha+1)}, \qquad p < 2,$$

$$u^\varepsilon(x) - u(x) \leqslant C\varepsilon^{2\gamma} + C\varepsilon + C\varepsilon|\log d(x)| + C\varepsilon^{1-\gamma}, \qquad p = 2.$$

If $p = 2$, the optimal choice of γ is given by $2\gamma = 1 - \gamma$, i.e., $\gamma = \frac{1}{3}$, which yields a rate of $\mathcal{O}(\varepsilon^{2/3})$, an improvement over the $\mathcal{O}(\sqrt{\varepsilon})$ estimate in Theorem 1.

If $p < 2$, by setting $2\gamma = (1-\gamma)(\alpha+1)$, we can get the best value of γ, that is, $\gamma = \frac{\alpha+1}{\alpha+3}$, and we obtain an improved estimate of $\mathcal{O}\left(\varepsilon^{\frac{2(\alpha+1)}{\alpha+3}}\right)$, noting that $\frac{2(\alpha+1)}{\alpha+3} = \frac{1}{p-1/2} > \frac{1}{p} > \frac{1}{2}$. ∎

References

[1] Han, Y., and Tu, S. N. T. Remarks on the Vanishing Viscosity Process of State-Constraint Hamilton–Jacobi Equations. *Applied Mathematics & Optimization 86*, 1 (June 2022), 3.